(iii) Angular speed ω

$v = wr$

$w = \frac{1.68}{0.5}$

$= 3.36 rads^{-1}$

(iii)

$(\uparrow) T\cos\theta = 0.5g$

$\cos\theta = 0.5g/10$

$\theta = 60.66^0$

Exercise

1. An object of mass 0.5kg on the end of the string is whirled around in a horizontal circle of radius 2m, with a constant speed of $10ms^{-1}$. Find its angular velocity and the tension in the string.(ω = $5rads^{-1}$, T = 25.5N)

2. A small ball of mass 0.1 kg is suspended by an inextensible string of length 0.5m and is caused to rotate in a horizontal circle of radius 0.4m. Find

(i) The resultant of these forces. (1.3N)

(ii) The period of rotation. (1.1s)

3. A pendulum bob of mass 0.2kg is attached to one end of an inelastic string of length 1.2m. The bob moves in a horizontal circle with the string inclined at 30° to the vertical. Calculate: (i) the tension in the string

(ii) the period of the motion

4. The period of oscillation of a conical pendulum is 2.0s. If the string makes an angle of 60° to the vertical at the point of suspension, calculate the:

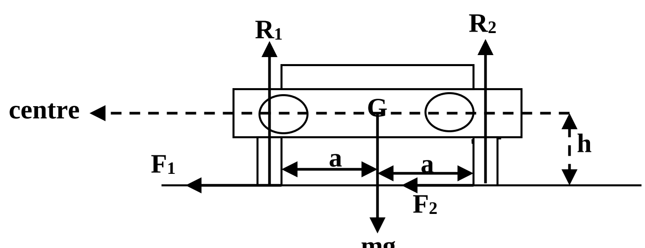
(i) Vertical height of the point of suspension above the circle.(h = 0.994m)

(ii) Length of the string, (l = 1.99m)

(iii) Velocity of the mass attached to the string.(v = $5.41ms^{-1}$)

<u>Vehicle on a curved track</u>

(i) Overturning / upsetting / toppling



consider a vehicle with mass *m* moving with a speed *v* in a circle of radius *r*; let *h* be the height of the centre of gravity above the truck and *2a* the distance between the tyres.

Resolving vertically :

$$R_1 + R_2 = mg .... (1)$$

horizontally

$$(F_1 + F_2) = \frac{mv^2}{r} .... (2)$$

Taking moments about G:

$$R_1 a + F_1 \times h + F_2 \times h = R_2 . a$$

$$(F_1 + F_2)\frac{h}{a} = R_2 - R_1 .... (3)$$

Substitute equation (2) in equation (3)

$$\frac{mv^2}{r} . \frac{h}{a} = R_2 - R_1 .... (4)$$

Add equation (1)+ equation(4)

$$R_1 + R_2 = mg$$

$$R_2 - R_1 = \frac{mv^2 h}{ra}$$

$$\therefore \frac{2R_2}{} = \frac{m}{}\left(g + \frac{v^2 h}{ra} \right)$$

$$R_2 = \frac{m}{2}\left(g + \frac{v^2 h}{ra} \right)$$

$R_2 > 0$ implying that the outer tire never lose contact.

Equation (1)– equation(4)

$$2R_1 = mg - \frac{mv^2 h}{ra}$$

$$R_1 = \frac{m}{2}\left(g - \frac{v^2 h}{ra} \right)$$

When $R_1 = 0$, inner tire loses contact with the track.

$$\Rightarrow \frac{m}{2}\left(g - \frac{v^2 h}{ra} \right) = 0$$

$$g - \frac{v^2 h}{ra} = 0$$

$$v^2 = \frac{rag}{h}$$

$$v = \sqrt{\frac{rag}{h}}$$

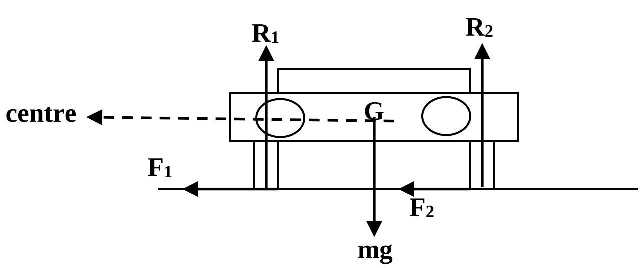
For speeds higher than $\sqrt{\frac{vag}{h}}$, the car overturns.

The vehicle is likely to overturn if

- ❖ The bend is sharp (r is small)
- ❖ The centre of gravity is high (h is large)
- ❖ The distance between the tires is small (a is small)

Skidding

A vehicle will skid when the available centripetal force is not enough to balance the centrifugal force (force away from the centre of the circle), the vehicle fails to negotiate the curve and goes off truck outwards.



For no skidding, the centripetal force must be greater or equal to the centrifugal force i.e.

$$F_1 + F_2 \geq \frac{mv^2}{r}$$

But $F_1 = \mu R_1$ and $F_2 = \mu R_2$

$$\mu(R_1 + R_2) \geq mv^2/r$$

$$\mu mg \geq \frac{mv^2}{r}$$

$$\mu g \geq \frac{v^2}{r}$$

$$v^2 \geq \mu gr$$

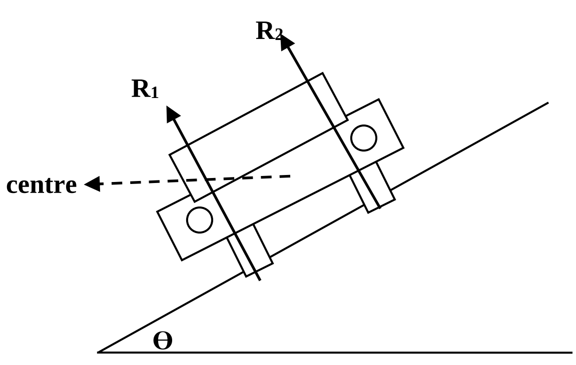
Maximum safe speed, $v_m = \sqrt{\mu rg}$

Skidding will occur if

- The vehicle is moving too fast
- The bend is too sharp (r is small)
- The road is slippery (μ is small)

BANKING OF A TRACK

- This is the building of the track round a corner with the outer edge raised above the inner one. This is done in order to increase the maximum safe speed for no skidding.
- When a road is banked, some extra centripetal force is provided by the horizontal component of the normal reaction
- When determining the angle of banking during the construction of the road, friction is ignored.



Resolving vertically

$$R_1\sin(90-\Theta) + R_2\sin(90-\Theta) = \frac{mv^2}{r}$$

But sin (90- Θ) = cos Θ

(R_1+R_2) Cos Θ = mg …………..(i)

Horizontally

$$R_1 \cos(90-\theta) + R_2 \cos(90-\theta) = \frac{mv^2}{r}$$

$$(R_1 + R_2)\sin\theta = \frac{mv^2}{r} ....(2)$$

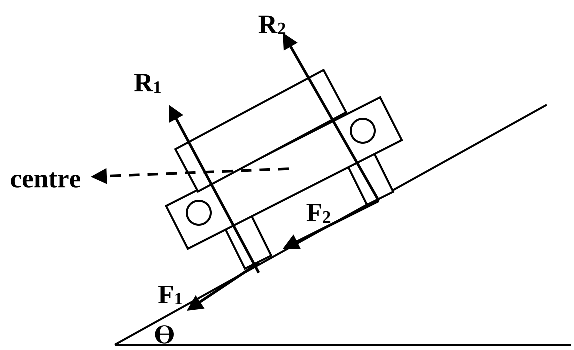
$$eqn\,2 \div \text{eqn}\,1$$

$$\tan\theta = \frac{\text{v}^2}{\text{rg}}$$

Hence Θ <u>is the angle of banking</u>

When there is friction

Suppose there is friction between the track and the vehicle moving round the bend.



Resolving vertically:

(R_1+R_2) cosΘ = (F_1+F_2)sin Θ +mg

(R_1+R_2) cos Θ – (F_1+F_2) sinΘ = mg

but $F_1 = \mu R_1$, $F_2 = \mu R_2$.

$(R_1+R_2)\cos\Theta - \mu(R_1+R_2)\sin\Theta = mg$

$(R_1+R_2)(\sin\Theta - \mu\sin\Theta) = mg$ ……………………..(1)

Horizontally

$$(R1 + R2)\sin\theta + (F1 + F2)\cos\theta = \frac{mv^2}{r}$$

$$(R1 + R2)\sin\theta + \mu(R1 + R2)\cos\theta = \frac{mv^2}{r}$$

$$(R_1 + R_2)(\sin\theta + \mu\cos\theta) = \frac{mv^2}{r} \text{...............}(2)$$

$$egn\ 2 \div \text{eqn}\,1$$

$$\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{rg}$$

$$\frac{\tan\theta + \mu}{1 - \mu\tan\theta} = \frac{v^2}{rg}$$

$$v^2 = rg\left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)$$

$$\therefore \text{maximum safe speed} = \sqrt{rg\left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)}$$

Question

(a) Why a rider has to bend at a certain angle when moving round a bend.

(b) Derive the angle of inclination the rider makes with the horizontal when moving round a bend.

2. A bend of 200m radius on a level road is banked at the correct angle for a speed of $15ms^{-1}$. If a vehicle rounds the bend at $30ms^{-1}$, what is the minimum co-efficient of kinetic friction between the tyres and the road so that the vehicle will not skid.

Angle of banking

$$= \tan\theta = \frac{v^2}{rg} = \frac{15^2}{(200\times 9.8)}$$

$$\theta = 6.55^0$$

$$v^2 = rg \left(\frac{\mu + \tan\theta}{1-\mu\tan\theta}\right)$$

$$30^2 = 200\times 9.8 \left(\frac{\mu + \tan\ 6.55}{1 - \mu \tan 6.55}\right)$$

$$900 = 1960 \left(\frac{\mu + 0.1148}{1 - 0.1148\mu}\right)$$

$$900 - 103.32\mu = 1960\mu + 225.008$$

$$2063.32\mu = 674.992$$

$$\mu = 0.327$$

2. A car travels round a bend in road which is a circular arc of radius 62.5m.

The road is banked at angle $\tan^{-1}\left(\frac{5}{12}\right)$ to the horizontal the coefficient of friction between the tyres of the car and the road surface is 0.4. Find

(i) the greatest speed at which the car can be driven round the bend without slipping.

(ii) The least speed at which this can happen.

(i) Maximum speed

$$v^2 = rg \left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)$$

$$v^2 = 62.5 \times 9.8 \left(\frac{0.4 + 5/12}{1 - 0.4 \text{x} 5/12}\right)$$

$$v^2 = 612.5\left(\frac{\frac{49}{60}}{5/6}\right)$$

$$v^2 = 600.25$$

$$v^2 = 24.5ms^{-1}$$

(ii) Least speed

$$v^2 = rg\tan\theta$$

$$v^2 = 62.5 \times 9.8 \times 5/12$$

$$v^2 = 255.208$$

$$v = 15.98ms^{-1}$$

Motion in a vertical circle

This is an example of motion in a circle with non- uniform speed. The body will have a radial component of acceleration as well as a tangential component. Consider a particle of mass is attached to an inextensible string at point O, and projected from the lowest point P with a speed U so that it describes a vertical circle.

Consider a particle at point Q at subsequent time.

The tension T in the string is everywhere normal to the path of the particle and hence to its velocity V. the tension therefore does no work on the particle.

Energy at P, E_P is E_p = ½mu^2 ……………………..(1)

P is the reference for zero potential . Energy at Q in E_q is:-

$E_q = ½mv^2 + mgh.$

But h = r-rcosΘ

Eq = $½ mv^2 + mgr (1-\cos\Theta)$ ………………….(2)

Centripetal force of the particle

$T- mg\cos\Theta = \frac{mv^2}{r}$

$Mv^2 = r (T-mg\cos\Theta)$ ………………………(3)

Substitute equation (3) into (2)

$E_q = ½ r (T- mg\cos\Theta) + mgr (1-\cos\Theta)$

Using conservation of mechanical energy

$E_q = E_p.$

$½r (T-mg\cos\Theta) + mgr (1-\cos\Theta) = ½mu^2$

$½r(T-mg\cos\Theta) = ½mu^2- mgr (1-\cos\Theta)$

$r(T-mg\cos\Theta) = mu^2 – 2mgr(1-\cos\Theta)$

$T-mg\cos\Theta) = \frac{mu^2}{r} - 2mg(1-\cos\theta)$

$$T = \frac{mu^2}{r} - 2mg(1-\cos\theta) + mg\cos\theta$$

$T = \frac{mu^2}{r} + mg(2\cos\theta + \cos\theta - 2).$

$T = \frac{mu^2}{r} + mg(3\cos\theta - 2).$

OR $\mathrm{T} = \frac{\mathrm{mu}^2}{\mathrm{r}} - mg(2 - 3\cos\theta)$

T is greater than zero when $\frac{mu^2}{r} + mg(\cos\theta - 2) > 0$

$\frac{\mathrm{mu}^2}{\mathrm{r}} > mg(2 - 3\cos\theta)$

$u^2 > rg(2 - 3\cos\theta)$

When $\vartheta = 90$

$\mathrm{u}^2 > rg(2 - 3\cos 90)$

$u^2 > 2rg$

Hence particle overshoots point 0' when u> $\sqrt{\ }$ 2rg

When Ɵ = 180^0

$u^2 > rg$ (2-3cos 180)

$u^2 > 5rg$

Hence particle reaches p' when U> $\sqrt{5rg}$

Therefore particle describes a circle when the initial speed with which you project from P is $u \geq \sqrt{5rg}$

Example

1. A cyclist rounds a curve of 30m radius on a road which is banked at an angle of 20^0 to the horizontal. If the co-efficient of sliding friction between the tires and the road is 0.5; find the greatest speed at which the cyclist can ride without skidding and find into inclination to the horizontal at this speed.

$$v^2 = rg\left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)$$

$$v^2 = (30 \times 9.8)\left(\frac{0.5 + \tan 20}{1 - 0.5\tan 20}\right)$$

$$v^2 = 294\left(\frac{0.1819}{0.818}\right)$$

$$v = 17.6ms^{-1}$$

$$\tan\theta = \frac{v^2}{rg} = \frac{17.6^2}{30x9.8}$$

$$\theta = 46.5°$$

4(b) A car goes round unbanked curve at 15ms^{-1}the radius of the curve is 60m. Find the least co-efficient of kinetic friction that will allow the car to negotiate the curve without skidding.

$$\mu \geq \frac{v^2}{r}$$

$$\mu \geq \frac{v^2}{rg}$$

$$\mu \geq \frac{15^2}{(60 \times 9.8)} = 0.38$$

Exercise

1. A stone of mass 0.5kg is attached to a string of length 0.5m which will break if the tension in it exceeds 20N. The stone is whirled in a vertical circle, the axis of rotation being at a vertical height of 1.0m above the ground. The angular speed is gradually increased until the string breaks.

(i) in what position is the string most likely to break?(vertically below point of suspension)

(ii) At what angular speed will the string break? (7.7rads^{-1})

(iii) Find the position where the stone hits the ground when the string breaks.1.22m from point below point of suspension)

2. A car travels round a curved road bend banked at an angle of 22.6°. If the radius of curvature of the bend is 62.5m and the coefficient of friction between the tyres of the car and the road surface is 0.3. Calculate the maximum speed at which the car negotiates the bend without skidding. ($22.4ms^{-1}$)