S.3 MATHEMATICS FOR SELF STUDY

FORMULAE

A formula is a mathematical sentence in which one quantity is expressed in terms of other letters or numbers and letters. For example,

1. Area of a rectangle = length × width, i.e. A = lw;
2. C = 2πr is a formula for finding the circumference, C, of a circle of radius r.

The single letter or quantity on the left hand side (LHS) is called the of the formula.

The subject of the formula must occur only once, isolated, on the LHS/RHS of any algebraic equation.

For example in the formula: v = u + at, v is the subject of the formula.

Change of the subject.

Frequently it is convenient to change the subject of a formula.

For example;

Consider the formula C = 2πr.

In the above form, the subject of the formula is C.

However, if we divide both sides of the formula by 2π:

$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$

$\therefore \frac{C}{2\pi} = r$ It follows that, the subject has been changed to r.

Example

Make I the subject in the formula, $T = P + I$.

Solution

$T = P + I$

Subtracting P from both sides gives,

$T - P = I$

∴ I = T – P.

Example

Make the bold letter the subject in each of the following formulae

a) $A = \frac{1}{2}h(\mathbf{a} + b)$

b) $x = \sqrt{\mathbf{A}}$

c) $V = \pi\mathbf{r}^2h$

d) $\frac{1}{\mathbf{p}} = \frac{1}{x} + \frac{1}{y}$

Solution

a) (a) $A = \frac{1}{2}h(\mathbf{a} + b)$

Multiplying both sides by 2

$2A = h(a + b)$

Expanding the right hand side:

$2A = ha + hb$

Subtracting hb from both sides:

$2A - hb = ha$

Dividing through the equation by h:

$\frac{2A - hb}{h} = a$

(b) $x = \sqrt{A}$

Square both sides of the equation

$x^2 = (\sqrt{A})^2$

$\therefore A = x^2$

(c) $V = \pi\mathbf{r}^2h$

Dividing through the equation by πh

$\frac{V}{\pi h} = r^2$

Taking square root on both sides of the equation:

$\sqrt{\frac{V}{\pi h}} = \sqrt{r^2}$

$\therefore r = \sqrt{\frac{V}{\pi h}}$

d) $\frac{1}{\mathbf{p}} = \frac{1}{x} + \frac{1}{y}$

Multiplying each term in the equation by LCM of all the denominators, pxy

$$xy = py + px$$

Factorizing the right hand side (RHS):

$$xy = p(x + y)$$

Dividing through the equation by $(x + y)$

$$\frac{xy}{x + y} = p \therefore$$

NOTE:

In general, to change the subject of a formula, i.e. to express one variable in terms of the other variable and/or numbers:

a) Remove fractions by multiplying each **term** in the equation by the LCM of all the denominators.
b) Arrange the terms containing the subject (required letter) on one side of the equation.
c) Factorize and divide by the coefficient of the subject (required letter).
d) Square both sides if the letter is under a square root sign and vice versa for a square on the variable
e) Do not alter capital letters to small letters or vice versa.

Example 7.3

The area and circumference of a circle are given by the formula $A = \pi r^2$ and

$C = 2\pi r$ respectively, Show that $C = 2\sqrt{\pi A}$

Solution

$A = \pi r^2$, hence $r^2 = \frac{A}{\pi}$ …………………(i)

$\rightarrow c^2 = 4\pi^2 r^2$………………(ii)

$C^2 = 4\pi^2 \frac{A}{\pi}$ … … … … … … … … … … … … (iii)

$C = \sqrt{4\pi A}$ ………………………..(iv)

$\therefore C = 2\sqrt{\pi A}$ as required.

Example

(a) If $a + \frac{bx}{c} = dx$, express x in terms of a, b, c and d.

(b) If $\frac{x+d}{c} = \frac{25d}{x-d}$, obtain x in terms of c and d.

Solution

(a) $a + \frac{bx}{c} = dx$

Multiplying each term in the equation by the LCM, c.

$ac + bx = cdx$

Collecting the terms with x together

$ac = cdx - bx$

Factorizing the right hand side

$ac = x(cd - b)$

$\therefore x = \frac{ac}{cd-b}$.

(b) $\frac{x+d}{c} = \frac{25d}{x-d}$,

Multiplying each term in the equation by the LCM of all the denominators,$c(x - d)$

$(x + d)(x - d) = 25cd$

$x^2 - d^2 = 25cd$

$x^2 = 25cd + d^2$

$\therefore x = \sqrt{25cd + d^2}$

EXERCISE:

1. Make **m** the subject in this formula **y = mx + c.**

2. Make **s** the subject in this formula

$$\boldsymbol{v^2 = u^2 + 2as.}$$

3. If $s = ut + \frac{1}{2}at^2$, express **a** in terms of **s, u** and **t.** Hence find **a** when **s = 19, u = 8** and **t = 2.**

4. Make **c** the subject in this formula $\frac{1}{c} = \frac{1}{a} + \frac{1}{b}$, hence find **c** when **a = 10** and **b = 15.**

5. If $T - mg = ma$, express **m** in terms of **T, g** and **a.** Hence find **m** when **T = 52, a = 3·2** and **g = 9·8.**

6. If $k(n+3) = 5n+2$, express **n** in terms of **k.** Hence find **n** when **k = 1.**

7. Make **v** the subject in this formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, hence find **v** when **f = 6** and **u = 10.**

8. If $\frac{1}{a} = \frac{bc}{b+c}$, make **c** the subject of the formula

9. Make **n** the subject in this formula

$$I = \frac{nE}{nR + r}$$

10. Make **V** the subject in this formula $R = \frac{Vr}{V-2}$. Hence find **V** when **R = 5r**

11. If $s = \frac{rk - a}{r - 1}$, make **c** the subject of the formula**.** Hence find **r** when **s = 93, a = 3** and **k = 48.**

12. If $V = \frac{1}{3}\pi r^2 h$, make **r** the subject of the formula**.**

13. If $a^2 + b^2 = c^2$, make **b** the subject of the formula**.** Hence find the values of **b** when **c = 10** and **a = 6.**

14. If $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, make **y** the subject of the formula**.** Hence find the values of **y** when **a = 4, b = 8** and **x = 5.**

15. If $b^2 = 4ac + (2ax + b)^2$, make **x** the subject of the formula**.** Hence find the values of **x** when **a = 2, b = –4** and **c = –6.**

16. Make **r** the subject in this formula $V = \frac{4}{3}\pi r^3$. Hence find **r** when **v = 38·808** and $\pi = \frac{22}{7}$.

17. Make **b** the subject in this formula $a = \frac{b^3}{b^3 + c}$. Hence find **b** when **c = 864** and **a = 0·2.**

18. Make **r** the subject in this formula

$$V = \frac{\pi p r^4}{8kl}.$$

19. Make **r** the subject in this formula

$$A = P\left(1 + \frac{r}{100}\right)^n,$$ hence find **r** when

A = 8,820, p = 8,000 and **n = 2.**

20. If $k = \sqrt{\frac{w}{w+a}}$, make **w** the subject of the formula

21. Make **d** the subject in this formula

$$p = 2\pi\sqrt{\frac{d}{d-k}}.$$

22. Make **k** the subject in this formula

$$p = \left(\frac{k-1}{k+1}\right)^{\frac{1}{2}}.$$

23. Make **k** the subject in this formula

$$p = \sqrt[3]{\frac{b(x-k)}{k}}.$$

23. Make **x** the subject in this formula

$$k = \sqrt[4]{px^2 - d}.$$

24. If $y = \frac{1}{2}mv^2$ and $k = \frac{x}{v}$, express **y** in terms of **m, x** and **k.**

25. If $V = \frac{1}{3}\pi r^2 h$ and $A = 4\pi r^2$, express **V** in terms of **A** and **h.**

<u>EXTRA EXERCISE:</u>

1. Make **a** the subject in this formula $v = u + at$. Hence find **a** when **v = 12t** and **u = 5t.**

2. If $c = \frac{5(f-32)}{9}$, express **f** in terms of **c**

3. Make **P** the subject in this formula $I = \frac{PRT}{100}$. Hence find **P** when **I = 740, T= 2** and **R = 5 .**

4. Make **s** the subject in this formula $v = \sqrt{u^2 + 2as}$. Hence find **s** when **v = 9, u= 5** and **a = 3·5 .**

5. Make **d** the subject in this formula

$$T = 2\pi\sqrt{\frac{d}{g}}.$$

6. If $\frac{p}{q} = \frac{x}{x+c}$, make **x** the subject of the formula

7. If $p = \frac{k(x-a)}{a}$, make **a** the subject of the formula

8. Make **V** the subject in this formula $R = \frac{Vr}{V-2}$. Hence find **V** when **R = 5r**

9. If $p = \frac{4q + r}{r}$,

(i) find the value of **p** when **q = r.**

(ii) make **r** the subject

10. If $p = \sqrt{\frac{k(x-a)}{a}}$, make **a** the subject of the formula

11. Make **k** the subject in this formula

$$p = \sqrt[3]{\frac{k-1}{k+1}}.$$

12. Make **y** the subject in this formula $a = \frac{x}{y^2} - m$. Hence find the values of **y** when **x = 80,** a = **2** and **m = 3.**

13. If $m = (Ax - B)^2$, make **x** the subject of the formula**.** Hence find the values of **x** when **m = 16, A = 2** and **B = 6.**

14. Make **r** the subject in this formula $A = P\left(1 + \frac{r}{100}\right)^4$, hence find **r** when

A = 20,736 and **P = 10,000.**

15. Make **b** the subject in this formula $h = \sqrt{\frac{ab^3}{a+b^3}}$. Hence find **b** when **h = 6** and **a = –108 .**

16. If $k = \sqrt{\frac{x+1}{x}}$, make **x** the subject of the formula

17. If $y = \frac{4x+3}{x+1}$, express **x** in terms of **y**

18. If $p = (1+x)^n$, express **x** in terms of **p** and **n**

END